VERSION **1**

McGILL UNIVERSITY
FACULTY OF SCIENCE
FINAL EXAMINATION

MATHEMATICS 140 2008 09 CALCULUS 1

EXAMINER: Professor W. G. Brown
ASSOCIATE EXAMINER: Dr. D. Serbin

DATE: Sunday, December 07th, 2008
TIME: 09:00 – 12:00 hours

FAMILY NAME:
GIVEN NAMES:

STUDENT NUMBER

Instructions

1. Do not tear pages from this book; all your writing — even rough work — must be handed in. You may do rough work for this paper anywhere in the booklet.
2. Calculators are not permitted. This is a closed book examination. Regular and translation dictionaries are permitted.
3. This examination booklet consists of this cover, Pages 1 through 7 containing questions; and Pages 8, 9, 10 and 11, which are blank. A TOTAL OF 70 MARKS ARE AVAILABLE ON THIS EXAMINATION.
4. You are expected to simplify all answers wherever possible.

- Most questions on this paper require that you SHOW ALL YOUR WORK!
  Solutions are to be begun on the page where the question is printed; a correct answer alone will not be sufficient unless substantiated by your work. You may continue your solution *on the facing page*, or on the last pages, or the back cover of the booklet, but you must indicate any continuation clearly on the page where the question is printed! *To be awarded partial marks on a part of a question a student's answer for that part must be deemed to be more than 50% correct.* Most of these questions will require that the answer be written in a box provided on the page where the question is printed; even if you continue your work elsewhere, the answer should be in the box provided.
- Some questions on this paper require only BRIEF SOLUTIONS ; for these you must write the correct answer in the box provided; you are not asked to show your work, and you should not expect partial marks for solutions that are not correct. Check your work!

PLEASE DO NOT WRITE INSIDE THIS BOX

1(a) /2	1(b) /2	1(c) /2	1(d) /2	1(e) /2	2(a) /3	2(b) /3	2(c) /3	3(a) /3	3(b) /3	3(c) /3	3(d) /3
4(a) /6	4(b) /4	5(a) /3	5(b) /3	5(c) /3	6 /10	7(a) /3	7(b) /3	7(c) /2	7(d) /2	TOTAL /70	

1. BRIEF SOLUTIONS

[2 MARKS EACH] Give the (simplified) numeric value of each of the following limits if it exists; if the limit is $+\infty$ or $-\infty$, write $+\infty$ or $-\infty$ respectively. In all other cases write "NO FINITE OR INFINITE LIMIT".

(a) When $f(x) = \tan 2x$, $\lim\limits_{x\to\frac{\pi}{8}} \dfrac{f(x) - f\left(\frac{\pi}{8}\right)}{x - \frac{\pi}{8}} =$

ANSWER ONLY

(b) $\lim\limits_{x\to 0} \dfrac{x - \sinh x}{x - \operatorname{sech} x} =$

ANSWER ONLY

(c) $\lim\limits_{x\to+\infty} \arcsin(-\cos x) =$

ANSWER ONLY

(d) $\lim\limits_{y\to 0^-} (1 - 15y)^{\frac{1}{y}} =$

ANSWER ONLY

(e) $\lim\limits_{x\to-\infty} \left(5 + \sqrt{x^2 + 4x + 5} + x\right) =$

ANSWER ONLY

2. BRIEF SOLUTIONS

[3 MARKS EACH] Answer each question.

(a) If $f(x) = \left\{ \begin{array}{ll} x^2 + bx & \text{if } x \leq 1 \\ cx + d & \text{if } x > 1 \end{array} \right\}$, find values of constants b, c, d such that f is differentiable and $f(2) = 3$.

ANSWER ONLY

(b) Determine values for constants L, M that will make

$$\lim_{x \to \infty} \left(\frac{x^2 + 1}{x + 1} - Lx - M \right) = 3 \, .$$

ANSWER ONLY

(c) A ladder 20 feet long rests against a vertical wall. If the bottom of the ladder is being pulled towards the wall at a rate of 1 ft./s, how fast, in ft/s, is the top of the ladder rising up the wall when the bottom of the ladder is 12 feet from the wall?

ANSWER ONLY

3. BRIEF SOLUTIONS

[3 MARKS EACH] Evaluate each of the following, and *always simplify your answers as much as possible.*

(a) The interval(s) where the graph of $f(x) = 2x^3 - x^4$ is concave downward is/are

ANSWER ONLY

(b) If $y = 2x \cosh 3x$, then the value of $\dfrac{dx}{dy}$ when $x = 0$ is

ANSWER ONLY

(c) An antiderivative $G(t)$ of $g(t) = \dfrac{-6}{\sqrt{1-t^2}}$ such that $G(0) = 12$ is

ANSWER ONLY

(d) Where $f(x) = \dfrac{(\ln x)^x}{x^{\ln x}}$, $f'(x) =$

ANSWER ONLY

4. SHOW ALL YOUR WORK!

(a) [6 MARKS] Use Rolle's Theorem and the Intermediate Value Theorem to show that the curve $y = 2x - 3\tan x + 1 \quad \left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$ crosses the x-axis exactly once.

(b) [4 MARKS] If it is known that a function f has derivative

$$f'(x) = (x+1)(x-4)^2(x+5)^3\,,$$

carefully determine which of the critical points of f are local extrema *using only the First Derivative Test*. Show all your work.

5. SHOW ALL YOUR WORK!

The equation $x + y = (x - y)^3$ defines y implicitly as a function of x near the point $(x, y) = (1, 0)$. Showing all your work

(a) [3 MARKS] determine the value of y' at $(x, y) = (1, 0)$; and

(b) [3 MARKS] determine the value of y'' at $(x, y) = (1, 0)$.

(c) [3 MARKS] Use a linear approximation to y' to estimate the slope of the tangent line to the curve with equation $x+y = (x-y)^3$ when $x = 0.95$.

The instructors are aware that you do not have the use of a calculator.

6. SHOW ALL YOUR WORK!

   [10 MARKS] A wooden box with a square base and a cover is constructed from thin panels of pine and oak; pine costs half as much as oak, and the back and bottom of the box are made of pine, with all other panels made of oak. Showing all your work, determine the dimensions of the cheapest box of volume 6 m^3, and the material to be used for each panel. (Marks assigned depend on your supplying a full, readable solution.)

7. SHOW ALL YOUR WORK!

When $x \neq 0$ define $f(x) = x \ln |x|$.

(a) [3 MARKS] Show that the discontinuity in the definition of f can be removed by defining $f(0) = 0$. For the rest of the problem, assume that $f(0) = 0$.

(b) [3 MARKS] Showing all your work, determine all local extrema of f, and classify them, if any, as maxima or minima; and all inflection points, if any.

(c) [2 MARKS] Showing all your work, find all horizontal and vertical asymptotes, if any. (It is not sufficient to simply list the asymptotes, if any.)

(d) [2 MARKS] Sketch the graph of f.

CONTINUATION PAGE FOR PROBLEM NUMBER

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